Phase Shifting Interferometry algorithms performance characterization under random stepping errors.

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Brief outlook

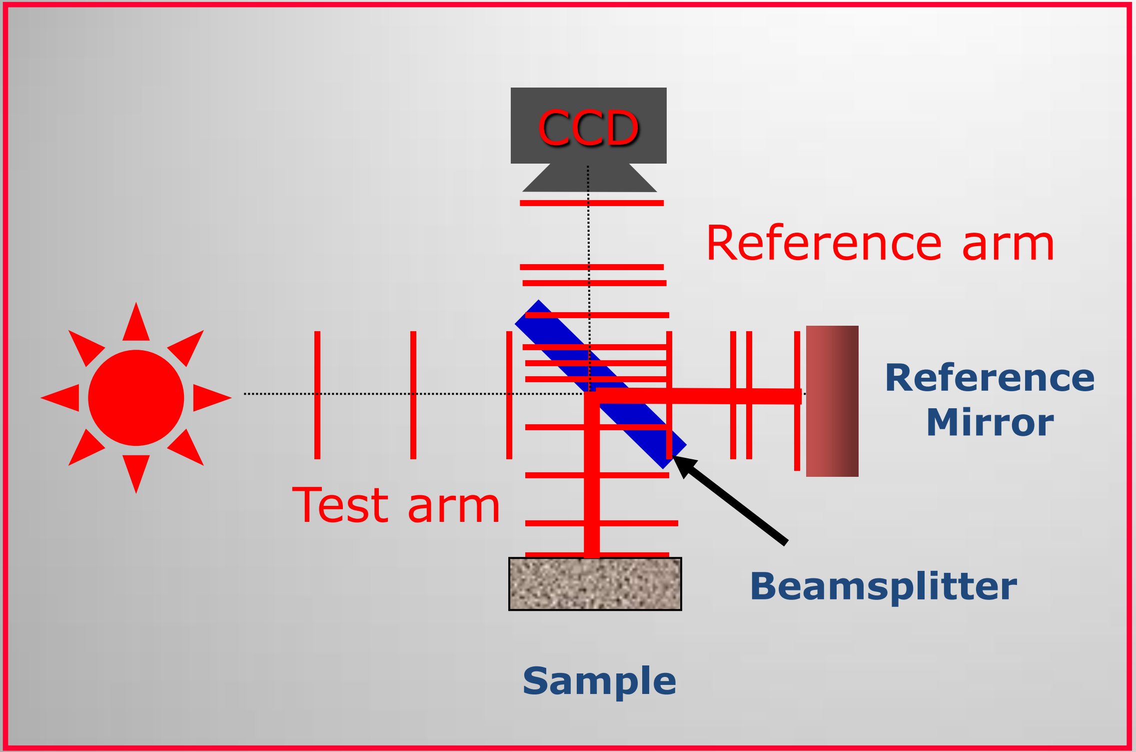
- Laser interferometry is a powerful technique:
  - based on phase estimation from successive measurements.
  - used:
    - at NIF by VISAR (Velocity Interferometer System for Any Reflector).
    - in industry for surface profiling
    - for stellar measurements (JPL).

- The accuracy depends on the precise positioning for data collection.

- Many parallel algorithms were developed to compensate for systematic positioning errors (bias), but no global characterization exists.

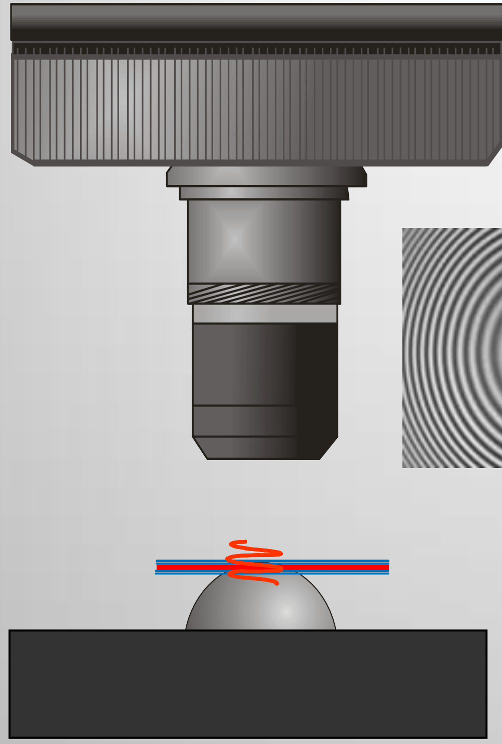
- A new quality measure for algorithms with <u>equivalent bias</u> performance and additional random stepping errors is suggested.

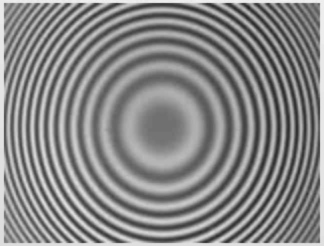
Optical interferometer – general principle



- The light from the source is split in two parts and sent on the object and on the reference mirror.
- The reflected beams are recombined.
- The recorded intensity depends upon the difference of the distances traveled by each beam -> constructive or destructive interference.

Phase shifting interferometry (PSI)





- Short scans, monochromatic light (λ).
- Frames are separated by an optical path difference (OPD) less than λ/2.

Distance measurement

- The recorded intensity by every CCD pixel is the combination of the reference "r" and object "o" beams.

$$I = I_o + I_r + 2\sqrt{I_o I_r}\, sin\left[\frac{2\pi}{\lambda}(z_r - z_o)\right]$$

- The reference (or object) position is varied in equal steps (Δ).

$$z_r^{(n)} = z_r^{(0)} + n\Delta$$

- The intensity at every position "n" is given by:

$$I_n = I_o + I_r + 2\sqrt{I_o I_r}\, sin\left[\frac{2\pi}{\lambda}(n\Delta) + \frac{2\pi}{\lambda}(z_r^{(0)} - z_o)\right]$$
$$\equiv A + B\sin(\omega_0 n + \phi)$$

- Distance measurement is a problem of phase detection.

Algorithms for phase detection

- A very large number designed to account for systematic stepping errors .
- Aim to determine a set of $\{a_n, b_n\}$ coefficients such that:

$$\tan(\phi) = \frac{\sum_n a_n I_n}{\sum_n b_n I_n} = \frac{\sum_n a_n \left[A + B\sin(\omega_0 n + \phi)\right]}{\sum_n b_n \left[A + B\sin(\omega_0 n + \phi)\right]}$$

- Necessary conditions:

$$\sum_n a_n = \sum_n b_n = 0$$

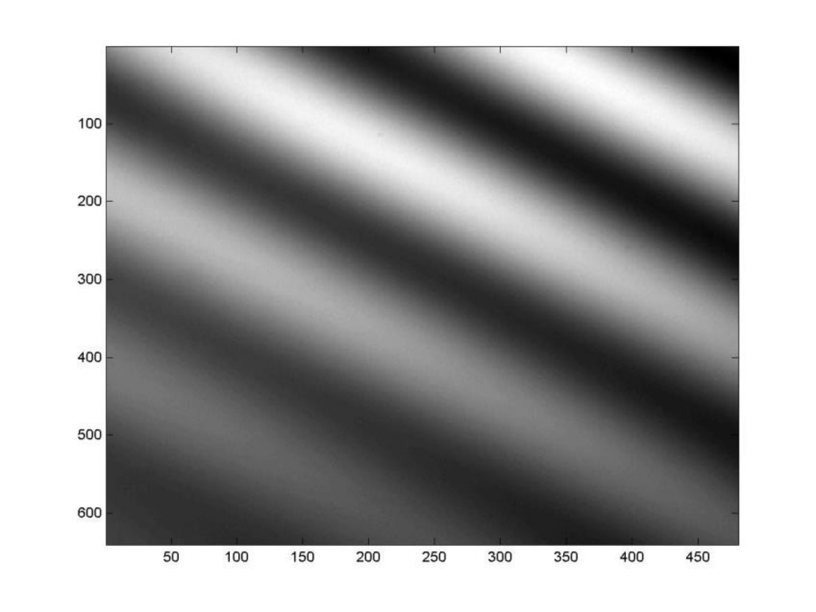
$$\frac{\sum_n a_n \sin(\omega_0 n + \phi)}{\sum_n b_n \sin(\omega_0 n + \phi)} = \frac{\sin(\phi + \zeta)}{\cos(\phi + \zeta)}$$

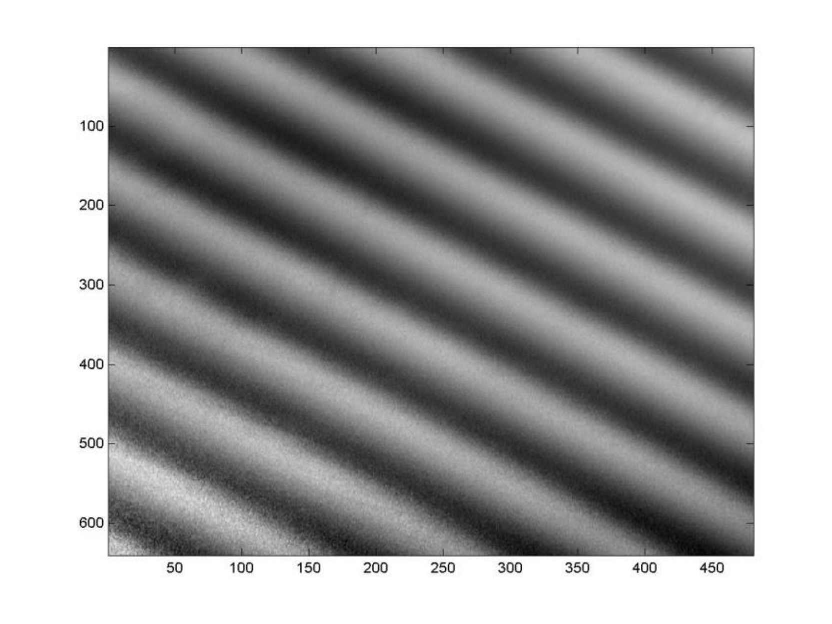
- Additional conditions can be imposed to compensate systematic uncertainties in ω.

Effects of stepping errors.

- Variations in steps (ω_0->ω_0 +δ) lead to errors in the measured phase (Bias).

$$\tan(\Delta\phi) = -\frac{r\sin(2\phi)}{1 + r\cos(2\phi)}$$





- One frame from a flat mirror.
- Recovered profile of a flat mirror.

Compensation of step errors – general.

- Specific errors are being addressed by using Fourier approach. Start by correlating the recorded signal I(z) with two unknown functions (f_1,f_2):

$$I(z) = \sum_{n=0}^{\infty} V_n cos(n\omega_0 z + \phi_n) \longrightarrow \begin{cases} p(z) = I(z) \oplus f_1(z) = \int_{-\infty}^{\infty} I(\zeta) f_1(z+\zeta) d\zeta \\ q(z) = I(z) \oplus f_1(z) = \int_{-\infty}^{\infty} I(\zeta) f_2(z+\zeta) d\zeta \end{cases}$$

$$r = \frac{p(0)}{q(0)} = \frac{\sum_{n=0}^{\infty} V_n \left[e^{i\phi_n} F_1^*(n\omega_0) + e^{-i\phi_n} F_1^*(-n\omega_0)\right]}{\sum_{n=0}^{\infty} V_n \left[e^{i\phi_n} F_2^*(n\omega_0) + e^{-i\phi_n} F_2^*(-n\omega_0)\right]}$$

- The phase corresponding to the frequency “m” can be retrieved from:

$$\tan(\phi_m) = r$$

if the following conditions are fulfilled:

$$F_1^*(n\omega_0) = F_2^*(n\omega_0) = 0, \qquad n \neq m$$

$$F_1^*(m\omega_0) = -iF_2^*(m\omega_0)$$

Compensation of stepping errors – example.

- Choose:

$$f_1(z) = \sum_{k=0}^{N} a_k \delta(z - z_k)$$

$$f_2(z) = \sum_{k=0}^{N} b_k \delta(z - z_k)$$

$$F_1^*(n\omega_0) = \sum_{k=0}^{N} a_k^* e^{i(n\omega_0)z_k}$$

$$F_2^*(n\omega_0) = \sum_{k=0}^{N} b_k^* e^{i(n\omega_0)z_k}$$

- Then the phase can be recovered from:

$$r = \frac{\sum_{k=0}^{N} a_k I(z_k)}{\sum_{k=0}^{N} b_k I(z_k)}$$

- A linear error in stepping corresponds to: $\omega_0 \to \omega_0 + \delta$

- In the case when the error is small:

$$F_1^*(n\omega_0) = \sum_{k=0}^{N} a_k^* e^{i(n\omega_0)z_k} \left[1 + (inz_k)\delta\right]$$

$$F_2^*(n\omega_0) = \sum_{k=0}^{N} b_k^* e^{i(n\omega_0)z_k} \left[1 + (inz_k)\delta\right]$$

- a set of additional equations for $\{a_n, b_n\}$ necessary to cancel (δ) can be derived.

Algorithm performance characterization.

- A very large number of algorithms have been derived.
- Their performance is currently estimated by the size of the phase error (bias) calculated from synthetic data with specific sampling errors.
- As of now, no way of characterizing algorithms with equivalent bias performance exists.

$$MSE(\hat{\phi}) \equiv E\left[(\hat{\phi}-\phi)^2\right] = \left[Bias(\hat{\phi},\phi)\right]^2 + Var(\hat{\phi})$$

- We suggest as a measure for the global performance the magnitude of the phase variance Var[Φ] in the presence of completely random stepping errors.
- Critical assumption: Correct steps -> Correct phase determination! (zero bias)

Stochastic analysis.

- Each data point in the signal is given by:

$$I_k = A + B\sin(k\Delta + \alpha_k + \phi) + \epsilon_k$$

$$E[\alpha_k] = \mu_\alpha \quad E[\epsilon_k] = 0$$

$$Cov[\alpha_k, \alpha_j] = \sigma_\alpha^2 \delta_{k,j} \quad Cov[\epsilon_k, \epsilon_j] = \sigma_\epsilon^2 \delta_{k,j} \quad Cov(\alpha_k, \epsilon_j) = 0$$

- Phase estimator:

$$\hat{\phi} = \arctan\left[\frac{\sum_{i=1}^{N} a_i I_i}{\sum_{j=1}^{N} b_j I_j}\right] - \zeta$$

- First order error propagation:

$$\mu_{\hat{\phi}} = E[\hat{\phi}] = E\left[\frac{\sum_{i=1}^{N} a_i I_i}{\sum_{j=1}^{N} b_j I_j}\right] \approx \left.\frac{\sum_{i=1}^{N} a_i E[I_i]}{\sum_{j=1}^{N} b_j E[I_j]}\right|_{\hat{\phi}=\phi}$$

$$\sigma_{\hat{\phi}}^2 = V[\hat{\phi}] \approx \sigma_\alpha^2 \sum_{k=1}^{N} \left[\frac{\partial\hat{\phi}}{\partial\alpha_k}\right]_{\hat{\phi}=\phi}^2 + \sigma_\epsilon^2 \sum_{k=1}^{N} \left[\frac{\partial\hat{\phi}}{\partial\epsilon_k}\right]_{\hat{\phi}=\phi}^2$$

Proposed estimator.

- Expected phase variance due to random step errors:

$$E_{\phi}^{step}\left[Var(\hat{\phi})\right] = \sigma_{\alpha}^{2}\left[\frac{1}{4(S_a^2+S_b^2)}\sum_{k=1}^{N}\left\{\begin{array}{c}(a_k^2+b_k^2)\\ +\sin(2\zeta)\left[-a_k b_k\cos(2k\Delta)+\frac{a_k^2-b_k^2}{2}\sin(2k\Delta)\right]\\ +\cos(2\zeta)\left[a_k b_k\sin(2k\Delta)+\frac{a_k^2-b_k^2}{2}\cos(2k\Delta)\right]\end{array}\right\}\right]$$

- Proposed measure for algorithm performance:

$$T(a_k, b_k, \Delta) \equiv \frac{1}{4(S_a^2+S_b^2)}\sum_{k=1}^{N}\left\{\begin{array}{c}(a_k^2+b_k^2)\\ +\sin(2\zeta)\left[-a_k b_k\cos(2k\Delta)+\frac{a_k^2-b_k^2}{2}\sin(2k\Delta)\right]\\ +\cos(2\zeta)\left[a_k b_k\sin(2k\Delta)+\frac{a_k^2-b_k^2}{2}\cos(2k\Delta)\right]\end{array}\right\}$$

$$S_a \equiv \sum_{k=1}^{N} a_k \sin(k\Delta) \qquad S_b \equiv \sum_{k=1}^{N} b_k \sin(k\Delta)$$

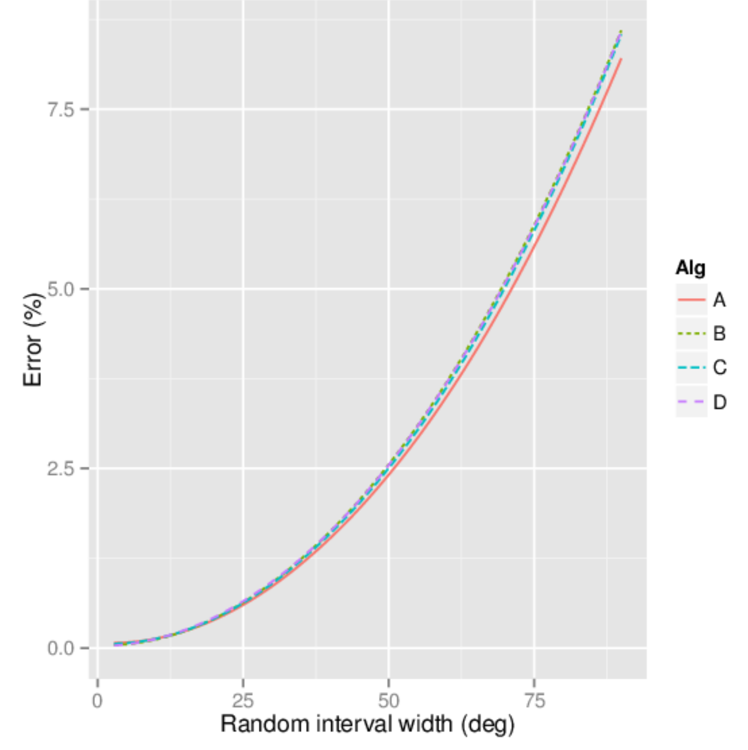
$$C_a \equiv \sum_{k=1}^{N} a_k \cos(k\Delta) \qquad C_b \equiv \sum_{k=1}^{N} b_k \cos(k\Delta)$$

Synthetic data results.

- Four algorithms were compared

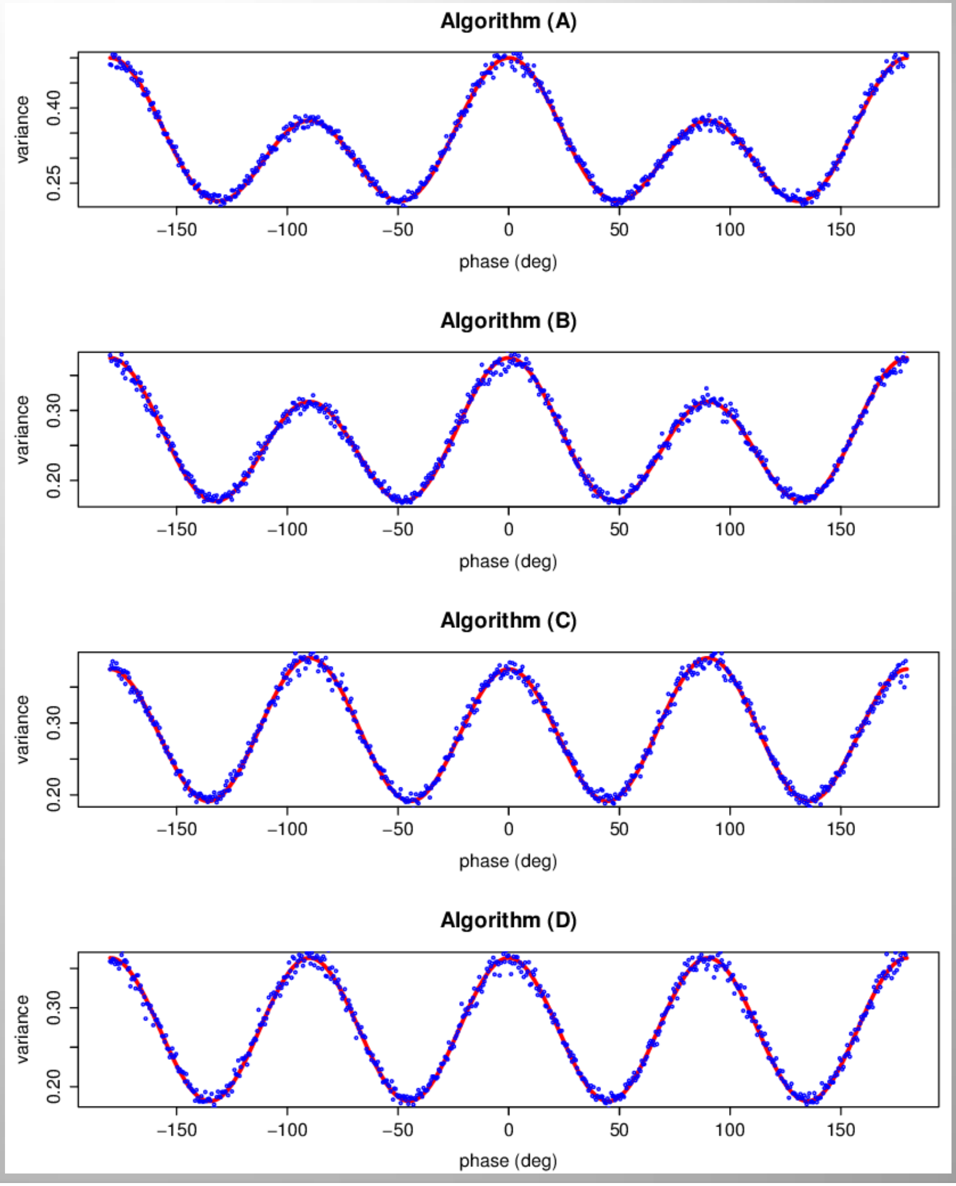
		Algorithm coefficients	
Name	No. frames	a	b
A	5	{0, -2, 0, 2, 0}	{1, 0, -2, 0, 1}
B	7	{0.5, 0, -1.5, 0, 1.5, 0, -0.5}	{0, 1, 0, -2, 0, 1, 0}
C	7	{1, 0, -7, 0, 7, 0, -1}	{0, 4, 0, -8, 0, 4, 0}
D	8	{1, -5, -11, 15, 15, -11, -5, 1}	{1, 5, -11, -15, 15, 11, -5, -1}

- Error (%) between the proposed formula and MC simulation with the size of the step error interval (in degrees).



Synthetic data results.

- Predicted (red line) and MC variances (blue dots) as function of signal phase.

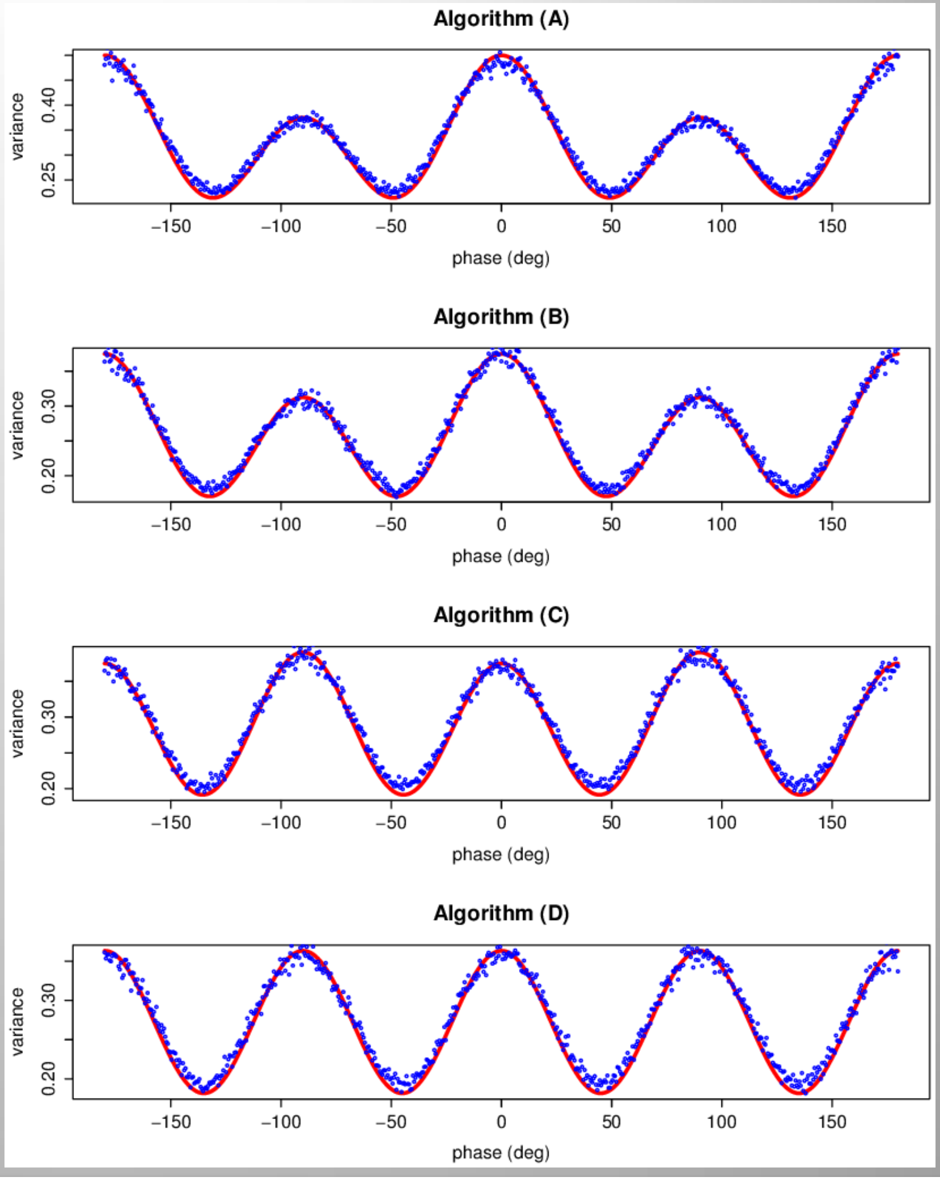


- Step error interval = π/10

Synthetic data results.

- Predicted (red line) and MC variances (blue dots) as function of signal phase.

- Step error interval = π/4



Algorithm performance comparison.

- No systematic error present.

Alg	No. frames (N)	$T(a_k, b_k, \Delta)$	$U(a_k, b_k, \Delta)$	$N \times T$	$N \times U$
A	5	0.328	0.437	1.64	2.187
B	7	0.258	0.344	1.80	2.41
C	7	0.287	0.383	2.01	2.68
D	8	0.272	0.363	2.18	2.91

- Linear mis-calibration step of 40 deg.

Alg	No. frames (N)	$T(a_k, b_k, \Delta)$	$U(a_k, b_k, \Delta)$	$N \times T$	$N \times U$
A	5	0.467	0.831	2.33	4.16
B	7	0.879	1.572	6.15	11.00
C	7	0.589	1.045	4.12	7.31
D	8	0.645	1.153	5.16	9.22

Conclusion.

- Selection of phase detection algorithm must be done based on:
  - Presence of known systematic errors.
  - Bias compensation performance.
  - Stochastic performance.

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